Direct and Large Eddy Simulations of Droplet Condensation in Turbulent Warm Clouds

G. Sardina, F. Picano, L. Brandt and R. Caballero

1 Introduction

A cloud is a complex multiphase system constituted by a huge number of different substances such as water droplets, ice droplets, water vapor, organic vapors, air. Warm clouds produce 30% of the total rainfall of the planet (70% at the tropics). The clouds are crucially important for Earth's climate, climate changes, rain formation and water cycle. Clouds are considered the largest source of uncertainty in climate prediction because it is very difficult to accurately parameterize the small-scale dynamics and turbulence effects (microphysical processes) affecting clouds in a global climate model where grid boxes used in simulations are typically 250 km wide and 1 km high while processes important for cloud formation happen at much smaller scales, ranging from meters to micrometers. Turbulence affects the cloud microphysics (ensemble of processes that create rain formation) by entrainment, stirring, and mixing, resulting in strong fluctuations and intermittency in temperature, humidity, aerosol concentration, and cloud particle growth and decay [1]. Nowadays a fundamental open question is the rapid growth of cloud droplets in the size range 15–40 μm in radius where the gravitational collision mechanism is not effective. This phenomenon is called condensation-coalescence bottleneck problem or size gap. Rain is activated when a wide droplet radius distribution is generated and turbulence is believed to accelerate this by creating an intermittent supersaturation field and promoting collisions modifying the local droplet concentration (particle inertial clustering) and relative velocity between colliding droplets (sling effect). These

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processes are difficult to capture because of the multi scale nature of the turbulent flows given by the high Reynolds number inside a cloud. Clearly at the moment, the complete turbulence/cloud dynamics are impossible to capture theoretically, in laboratory experiments, Direct Numerical Simulations (DNS) and direct measurements inside real clouds because of the limited resolution and control of the experimental devices. Here we will analyze the impact of turbulence on the droplet growth by water vapor condensation a possible key to understand the bottleneck problem. In the last 15 years, the increase of the computational capabilities allowed the first DNS simulations of turbulence/cloud interactions where individual droplets experience different supersaturations [2, 3] but up to now the results have been underestimated for the difficulties to run simulations up to 20 min, typical time of rain formation.

2 Methodology

The warm cloud that we want to investigate is up to order of $L = 100\,\mathrm{m}$ so that we can initially neglect the effects of spatial inhomogeneity and large scale variations of the thermodynamic parameters. Typical value of the turbulent kinetic energy dissipation ε is order $10^{-3}\,\mathrm{m}^2\,\mathrm{s}^{-3}$ so that the cloud Kolmogorov scale $\eta = (\nu/\varepsilon)^{1/4} = 1\,\mathrm{mm}$ where ν is the air kinematic viscosity. Under these conditions the flow field $\mathbf{u}$ can be described by the incompressible Navier–Stokes equations in homogeneous isotropic configurations and the supersaturation field s is transported by the fluid according to the Twomey model, as described in [3]:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \mathbf{f_u} \tag{1}$$

$$\frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s = \kappa \nabla^2 s + A_1 w - \frac{s}{\tau_s} \tag{2}$$

where p is the pressure, ρ is the air density, $\mathbf{f_u}$ is an external forcing able to maintain a statistically stationary state, κ is the diffusivity of water vapor in air, w is the third component of the velocity field acting in the gravity direction, $A_1 w$ is source/sink term of supersaturation resulting from the variation in temperature and pressure with height. The parameter τ_s is the relaxation time scale of the supersaturation field and depends on droplet concentration and dimensions [4] $\tau_s^{-1} = 4\pi \rho_w A_2 A_3 \sum R_i / V$ where R_i are the radii of the droplets in the volume V, ρ_w is the water density, A_1, A_2 and A_3 are functions of thermodynamic quantities. The droplets are evolved in a Lagrangian framework: under the hypothesis of small spheres with dimensions smaller than Kolmogorov scale, low mass fraction to neglect two-way coupling effects on the carrier flow, the only forces acting on the droplets are gravity and the Stokes drag:

Table 1 Parameters of the simulations. The resolution is N, the cloud size L_{box}, the root mean square of the turbulent velocity fluctuations v_{rms} and Taylor Reynolds number Re_λ

Label	N^3	L_{box} [m]	v_{rms} [m/s]	Re_λ
DNS A1/2	64^3	0.08	0.035	45
DNS B1/2	128^3	0.2	0.05	95
DNS C1/2	256^3	0.4	0.066	150
DNS D1	1024^3	1.5	0.11	390
LES E1	512^3	100	0.7	5000

$$\frac{d\mathbf{v_d}}{dt} = \frac{\mathbf{u}(\mathbf{x_d}, t) - \mathbf{v_d}}{\tau_d} - g\mathbf{e_z} \tag{3}$$

$$\frac{d\mathbf{x_d}}{dt} = \mathbf{v_d} \tag{4}$$

$$\frac{dR_i}{dt} = A_3 \frac{s(\mathbf{x_d}, t)}{R_i} \tag{5}$$

where $\mathbf{x_d}$ is the droplet position, $\mathbf{v_d}$ is the droplet velocity, $\mathbf{u}(\mathbf{x_d}, t)$ is the fluid velocity at droplet position, $\tau_d = 2\rho_w R_i^2/(9\rho\nu)$ is the droplet relaxation time, g is the gravity acceleration and $s(\mathbf{x_d}, t)$ the supersaturation at droplet position.

We performed different simulations by gradually increasing the size of the computational clouds from few centimeters to 100 m by keeping constant the small scales. We used both Direct Numerical Simulations (DNS) for the smaller clouds and a Large Eddy Simulation with a classic Smagorinsky model for the subgrid stress tensor. The governing Eqs. (1)–(5) are solved with a classical pseudo-spectral code for the fluid phase coupled with a Lagrangian algorithm for the droplets. All cases share the same turbulent kinetic energy dissipation $\varepsilon = 10^{-3}\mathrm{m}^2\,\mathrm{s}^{-3}$, a value typically measured in stratocumuli. This corresponds to the same small-scale dynamics, with Kolmogorov scale $\eta = (\nu^3/\varepsilon)^{1/4} \approx 1$ mm, Kolmogorov time $\tau_\eta = (\nu/\varepsilon)^{1/2} \approx 0.1$s and velocity $v_\eta = \eta/\tau_\eta \approx 1$ cm/s. We examine droplets with 2 different initial radii, 13 and 5 μ m, denoted as case 1 and 2, with supersaturation relaxation time $\tau_s = 2.5$ and 7s, and same concentration (130 droplets per cm^3). The reference temperature and pressure are $T = 283$ K and $P = 10^5$ Pa, with $A_1 = 5E - 4\ \mathrm{m}^{-1}$, $A_2 = 350\ \mathrm{m}^3$/kg, $A_3 = 50\ \mu\ \mathrm{m}^2$/s. The simulation parameters are reported in Table 1. Note that simulation DNS D1 represent the largest Direct Numerical Simulation of a warm cloud up to now. For computational reasons, the cases DNS C2 and DNS D1 have been integrated with a computational time less than 20 min.

3 Results

A sketch of the simulation is shown in Fig. 1. The computational domain represents a portion of a warm cloud in correspondence of its core region. In the cloud core, the turbulence can be approximated as homogeneous and isotropic and the effects of

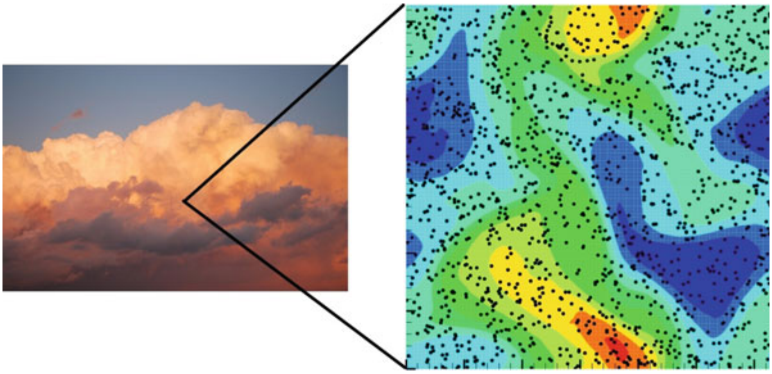


Fig. 1 Sketch of the simulations. The contours represent the supersaturation fluctuations while the discrete *dots* are the droplet positions

entrainment of dry air from the cloud boundaries can be neglected. The plot show a slice of the computational domain where the contours represents the supersaturation fluctuation field. The position of the droplets are superimposed to the contours, since the droplet relaxation time is much smaller than the Kolmogorov time, the droplets tend to be randomly distributed across the domain without preferential accumulation in specific flow regions.

In order to quantify the droplet spectral broadening, we will analyze the behavior of the root main square of the square droplet radius fluctuations indicated as $\sigma_{R^2} = \sqrt{\langle (R^{2'})^2 \rangle}$ where the symbol $\langle \cdot \rangle$ indicates the ensemble average between all the droplets in a given configuration and the apostrophe indicates the fluctuation respect to the mean value $\langle R^2 \rangle$ that in all cases is always equal to its value at time zero $\langle R_0^2 \rangle$ since the mean supersaturation is always zero $\langle s \rangle = 0 \Longrightarrow s = s'$.

The behavior of σ_{R^2} is shown in a log-log plot for all the DNS simulations (Fig. 2) and LES (Fig. 3). From the graphs we can observe for the first time in literature some important issues: (1) the most important is that the droplet spectral distribution never reach a statistical steady state but its rms continues always to increase due to turbulent fluctuations even if s has reached its quasi-steady state value s_{qs}. This implies that at infinite times all the clouds will precipitate because a sufficient spectral broadening will be reached unless external factors will influence the global cloud dynamics; (2) the rms increases according to a power law with exponent 0.5 for all the simulations independently from the Reynolds number; (3) the spectral broadening increases with the Reynolds number and so with the large/small scale separation, confirming previous results [3] where just the large scale turbulent fluctuations have a dominant role on the droplet spectral broadening. These behaviors can be easily explained by writing Eq. (5) for the square radius fluctuations, multiplying for itself and averaging:

$$\frac{d\langle (R_i^{2'})^2 \rangle}{dt} = \frac{d\sigma_{R^2}^2}{dt} = 2A_3 \langle s' R^{2'} \rangle \quad (6)$$

Fig. 2 Root main square of the square droplet radius fluctuations σ_{R^2} versus time for the DNS simulations. *Inset* correlation $\langle s' R^{2'} \rangle$ versus time

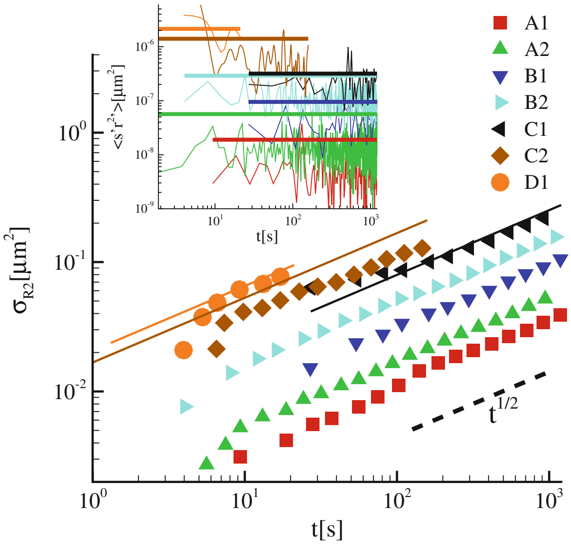
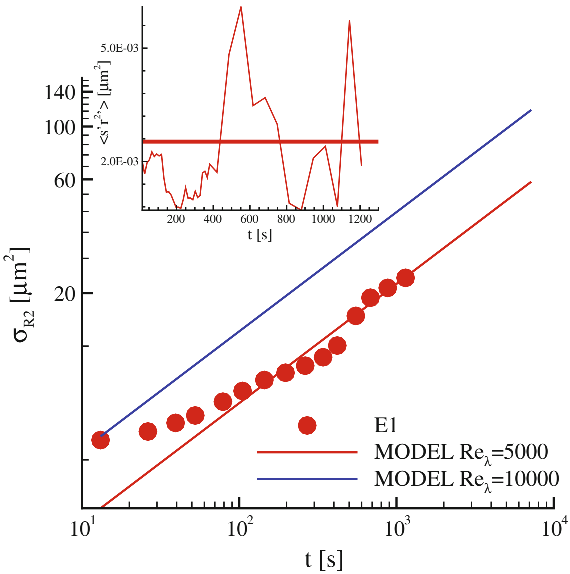


Fig. 3 Root main square of the square droplet radius fluctuations σ_{R^2} versus time for the LES simulations. *Inset* correlation $\langle s' R^{2'} \rangle$ versus time



the droplet square radius variance can linearly increase with time and consequently its rms can grow as $t^{1/2}$ if and only if the correlation $\langle s' R^{2'} \rangle$ reaches a temporal steady state. The previous correlation has been measured in the simulations and the results are plotted in the insets of Figs. 2 and 3. In all the cases the values of $\langle s' R^{2'} \rangle$ reach a statistical steady state fluctuating around a constant value that fixes the slope of σ_{R^2}. Essentially the role of turbulence in droplet condensation/evaporation is to create a constant correlation between supersaturation and square radius fluctuations that increases by increasing the scale separation and consequently the cloud sizes. LES results give us a value of σ_{R^2} order 25 μ m^2 after 20 min a value more close to the experimental observations found in stratocumuli. A 1-D stochastic model can be derived to compare the numerical results and predict the behavior at larger Reynolds number. The complete stochastic model is described in [5]. In particular the stochastic model predicts the value of the correlation $\langle s' R^{2'} \rangle$ and consequently of σ_{R^2}:

$$\langle s' R^{2'} \rangle_{qs} = 2A_3 A_1^2 v_{rms}^2 \langle \tau_s \rangle^2 T_0 = 2A_3 \langle s'^2 \rangle_{qs} T_0 \tag{7}$$

$$\sigma_{R^2} = \sqrt{8} A_3 A_1 v_{rms} \langle \tau_s \rangle (T_0 t)^{1/2} = 0.7 A_3 A_1 \nu^{1/2} \langle \tau_s \rangle Re_\lambda t^{1/2} \tag{8}$$

where T_0 is the turbulence integral time scale. The previous expression shows that the droplet/turbulence condensation dynamics does not depend on the turbulent small scales but is determined by the large flow scales. The results of the model are plotted in Fig. 2 in solid lines, the agreement improves by increasing the Reynolds number of the simulation and it is completely superimposed to the LES case. With the model prediction at higher Reynolds number can be obtained as it is shown in Fig. 3 for a case with $Re_\lambda = 10{,}000$.

4 Conclusions

We performed state of the art Direct and Large Eddy numerical simulation to study the impact of turbulence on cloud droplet growth by condensation. We show that the variance of the droplet size distribution increases indefinitely in time as $t^{1/2}$, with growth rate proportional the large-to-small turbulent scale separation and to the turbulence integral scales but independent of the smallest turbulence scale. Our results give finally a correct estimation of the impact of turbulence on the rain droplet condensation mechanism previously highly underestimated.

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